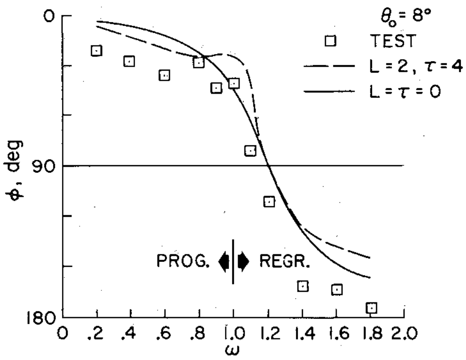

Fig. 4 Phase difference vs frequency, $\theta_0 = 8°$.

wake the velocities are $2(v_0 + v \sin\psi)$. The air, participating in the acceleration $\dot{v} \sin\psi$ is assumed to be enclosed in a cylinder with radius R and height h. The moment of momentum equation about the longitudinal axis contains only terms with factor $\sin^2\psi$ and reads

$$-\int_0^{2\pi}\int_0^R r d\psi dr \cdot r \sin\psi \rho(4v_0 v \sin\psi + h\dot{v} \sin\psi) = C_l(\Omega R)^2 \rho\pi R^2 \quad \text{(A1)}$$

where C_l is the aerodynamic hub rolling moment coefficient, positive to right. Performing the integrations and using nondimensional velocities $\lambda_0 = v_0/\Omega R$, $\lambda_{II} = v/\Omega R$ and the time unit $1/\Omega$, one obtains

$$\lambda_{II} + \tau\dot{\lambda}_{II} = -3C_l/4\lambda_0 \quad \text{(A2)}$$

where

$$\tau = h/4\lambda_0 R \quad \text{(A3)}$$

Performing the same analysis with a radially linear inflow velocity distribution assumed in Eqs. (1) and (2), one obtains

$$\lambda_{II} + \tau\dot{\lambda}_{II} = -5C_l/16\lambda_0 \quad \text{(A4)}$$

where

$$\tau = 5h/16\lambda_0 R \quad \text{(A5)}$$

In either case the expression $\lambda_{II} + \tau\dot{\lambda}_{II}$ is proportional to the left aerodynamic hub rolling moment, which is expressed in Eq. (7). Since τ and L are determined by correlation with tests, Eqs. (6) and (7) do not assume any specific inflow distribution over the radius.

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Dynamics of Slung Bodies Utilizing a Rotating Wheel for Stability

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Nomenclature

a,h	= horizontal and vertical distances between attachment points
d_1,d_2	= horizontal and vertical distances from the center of mass of the box to the cable attachment point along x, y, z
A,B,C	= moments of inertia of rectangular cargo container about the x, y, z axes
A_1,C_1,A_1	= moments of inertia of wheel about x_1, y_1, z_1 axes
D/W	= drag-to-weight ratio
l	= cable length
L,M,N	= aerodynamic moments
m_T	= mass of towed system
m_1	= mass of wheel
r	= wheel radius
R_1	= vertical distance from c.m. box to c.m. wheel along z
T_0	= steady-state cable force
u,v,w	= linear perturbation velocities
U_0	= x component of steady state velocity
W_0	= z component of steady state velocity
W_1/W	= wheel weight to system weight ratio
ω_1	= wheel rotational speed
X,Y,Z	= aerodynamic forces
θ,ψ,ϕ	= aircraft Euler angles
α	= angle of attack
α_0	= steady state angle of attack
β	= side-slip angle
C_D	= drag coefficient
C_L	= lift coefficient
C_Y	= side-force coefficient
C_l	= roll moment coefficient
C_m	= pitch moment coefficient
C_n	= yaw moment coefficient
C_{L_α}	= $\partial C_L/\partial\alpha$
C_{m_α}	= $\partial C_m/\partial\alpha$
C_{Y_β}	= $\partial C_y/\partial\beta$
C_{n_β}	= $\partial C_n/\partial\beta$
X_u, X_w, etc.	= changes in the aerodynamic forces and moments due to changes in velocities

Introduction

IN the past few years airborne towing has proven to be very useful for industrial and military transportation. Even though this means of transportation has demonstrated its effectiveness, reports have revealed that quite often serious instabilities have occurred. Asseo and Erickson[1] mention the dangerous load oscillations experienced while towing low density, high drag loads, which have resulted in emergency load jettison and some load/helicopter collisions. Similarly Etkin and Mackworth[2] report of serious instabilities which occurred while transporting loads of dense material in a specially designed bucket. Experimental investigations by Shanks[3-5] showed that lateral instability may arise in towing parawing gliders and half-cone re-entry vehicles. These problems have resulted in a number of investigations to determine the criteria necessary to insure stability during airborne towing.

Received June 18, 1973. This work was supported by the U.S. Army Research Office—Durham.

Index categories: Air Transportation Systems; VTOL Handling, Stability, and Control.

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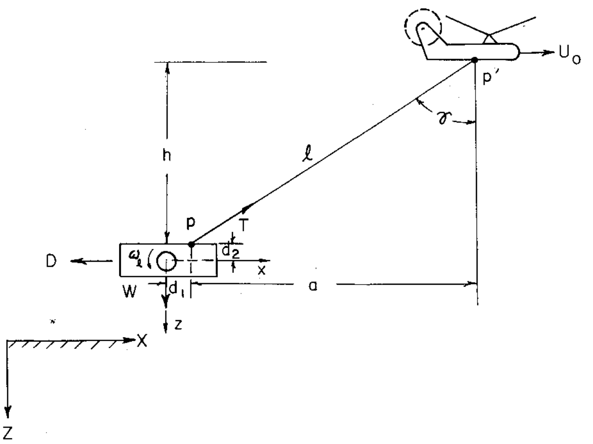

Fig. 1 Steady-state towing.

In a recent paper by Poli and Cromack[6] it is shown that long cables, high speeds, and light loads are required for the stability of a slung load using a single-point suspension system. The drag-to-weight ratio of the towed body and the cable length were shown to be the most important stability parameters. The towed body analyzed was an 8 × 8 × 20 ft cargo container. Cable lengths required for stability were found to range in excess of 800 ft at a drag-to-weight ratio of 0.01 to about 100 feet at a drag-to-weight ratio of 0.1.

Szustak and Jenny[7] discussed the use of multicable suspension systems. They showed that for the two and four-point suspension systems, short cables, low speeds and heavy loads are required for stability. It should be noted that these results are opposite to those obtained for a single-point suspension system, the reason being that for a single-point system it is mainly the drag force that aids in stability, whereas for the multipoint system it is the restoring moment of the cables which provides the stabilizing effect.

Attempts have also been made to actively stabilize slung loads. Poli and Cromack[6] show that the addition of stabilizing fins permits a reduction in cable length of 30% to 50% for a drag-to-weight ratio above 0.02.

Another method for stabilization was analyzed by Asseo and Erickson[1] in which they attempted to show the feasibility of using winches as active controllers for load stabilization. A three-point suspension system consisting of longitudinally and laterally displaced cables driven by vertical winches placed at the bottom of the helicopter structure was proposed. The front helicopter-cable attachment was considered capable of being laterally displaced relative to the helicopter. Various cable lengths and cargo orientations could thus be obtained such that stability could be insured. While the method was shown to be feasible, the major problem appears to be the complexity of the control system.

In many of the previous works results have shown that it is difficult, costly, and sometimes not feasible to insure complete stability of the towed system. Further investigations are required to determine an effective means of achieving stability. For this reason, this note examines the use of a reaction wheel to aid in the stabilization of the slung load. Both the longitudinal and lateral degrees of freedom are considered.

Equations of Motion for Uncoupled State

The linearized equations of motion for a rigid body containing a rotating wheel and towed beneath an aircraft are derived in Ref. 8. These relationships are determined for three different wheel orientations. It is shown that for the particular wheel orientation, illustrated in Fig. 1 the longitudinal and lateral equations are uncoupled. This will be the case of interest here.

In deriving the equations of motion, several assumptions were made: 1) the towing craft is flying straight and level, 2) the aerodynamic forces and moments are such that the lateral and longitudinal degrees of freedom can be uncoupled, 3) the distance from the center of mass of the box to the center of mass of the box-wheel system is small, 4) the rotating wheel is initially spun up, 5) the housing unit of the wheel will have negligible mass and momentum, 6) friction in the wheel bearings is negligible, and 7) rigid body motion is uncoupled from the dynamical motion of the cable. The linearized equations of motion for this case are[8]

Longitudinal Equations of Motion

$$\begin{bmatrix} m_T\lambda - X_u & -X_w & -a/l & -Z_0 & -X_0/a & 0 \\ -Z_u & m_T\lambda - Z_w & h/l & -m_T U_0\lambda + X_0 & 0 & -X_0/a \\ M_u & M_w & 0 & -(B + C_1)\lambda^2 & 0 & 0 \\ -1 & 0 & 0 & 0 & \lambda & 0 \\ 0 & -1 & 0 & U_0 & 0 & \lambda \\ 0 & 0 & 0 & 0 & -a & h \end{bmatrix} \begin{bmatrix} u \\ w \\ \Delta T \\ \theta \\ x' \\ z' \end{bmatrix} = 0 \qquad (1)$$

Lateral Equations of Motion

$$\begin{bmatrix} m_T\lambda - Y_v & m_T U_0\lambda - X_0 & Z_0 & -X_0/a \\ -L_v & -\lambda\omega_t C_1 & \lambda^2(A + A_1) & 0 \\ -N_v & \lambda^2(C + A_1) & \lambda\omega_t C_1 & 0 \\ -1 & -U_0 & 0 & \lambda \end{bmatrix} \begin{bmatrix} v \\ \psi \\ \phi \\ y' \end{bmatrix} = 0 \qquad (2)$$

where $\lambda = d/dt$ and where the remaining symbols are defined in the Nomenclature. These equations, are only true for the rotating wheel and cable attachment point coinciding with the center of mass of the rectangular container. Since the polar moment of inertia of the wheel C_1 is negligible compared to the moment of inertia of the box B it will be neglected in the analysis to follow.

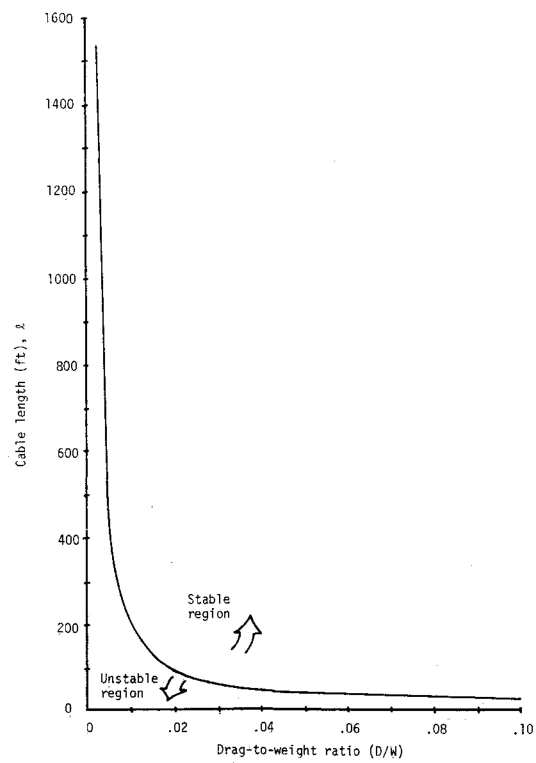

Fig. 2 Longitudinal stability of box with and without wheel.

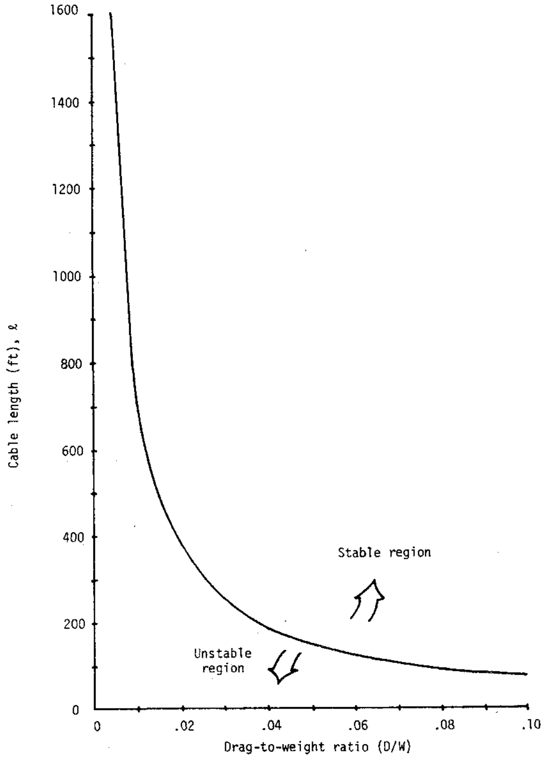

Fig. 3 Lateral stability of box alone.

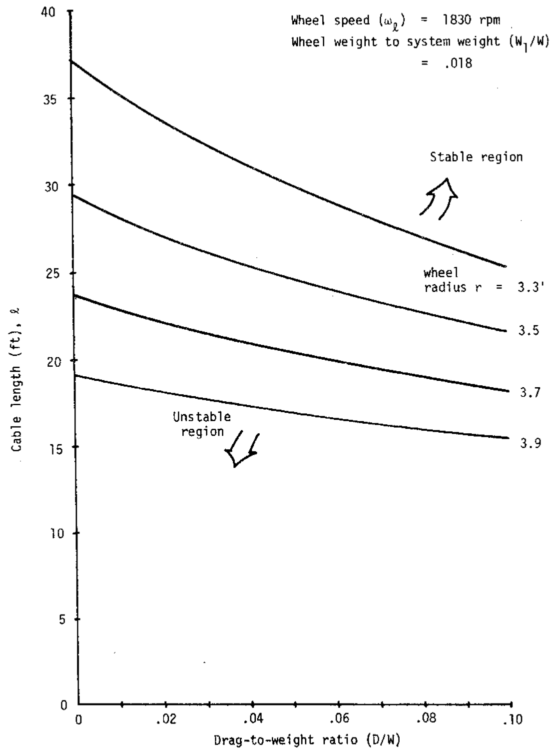

Fig. 4 Lateral stability of system, varying wheel size.

Stability Analysis

To perform the stability analysis, it is first necessary to expand the determinant of the coefficients in Eqs. (1) and (2), to obtain the characteristic equations. Performing the expansions gives:

Longitudinal characteristic equation

$$m_T B l^2 \lambda^4 - (a^2 B Z_w + h^2 B X_u)\lambda^3 - \left(l^2 m_T U_0 M_w + \frac{l^2}{a} B X_0\right)\lambda^2 + (h^2 U_0 X_u M_w + a^2 M_w X_0)\lambda + \frac{l^2}{a} X_0 U_0 M_w = 0 \qquad (3)$$

Lateral characteristic equation

$$m_T(C + A_1)\lambda^4 - Y_v(C + A_1)\lambda^3 + \left[-X_0 \frac{(C + A_1)}{a} + N_v m_T U_0 + \frac{\omega_l^2 C_1^2 m_T}{A + A_1}\right]\lambda^2 - \left(N_v X_0 + \frac{\omega_l^2 C_1^2 Y_v}{A + A_1}\right)\lambda - \left(\frac{N_v U_0 X_0}{a} + \frac{\omega_l^2 C_1^2 X_0}{a(A + A_1)}\right) = 0 \qquad (4)$$

Since the previous equations apply when the cable and the wheel are at the center mass of the box, steady state equilibrium conditions occur when $\alpha = \beta = 0$, which results in $Z_0 = Z_u = M_u = L_v = 0$.[6] For equations of the form,

$$a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$$

a necessary and sufficient condition for stability is that all the coefficients of λ be positive, and that

$$a_3(a_1 a_2 - a_0 a_3) - a_1^2 a_4 > 0 \qquad (5)$$

Using the aerodynamic data provided in Ref. 6, it is seen that in Eqs. (3) and (4) all the coefficients are positive, thus, all that is required for stability is that Eq. (5) be satisfied. Substituting the appropriate values of $a_0 \ldots a_5$ from Eqs. (3) and (4) into Eq. (5) gives the following inequalities, for an 8 × 20 ft container:

Longitudinal stability equation

$$l > \frac{-(1.933\, C_D + 110.9\, C_{L_\alpha})(D/W)^2 - 3.866\, C_D}{57.3\, C_{m_\alpha}[(D/W)^2 + 2](D/W)} \qquad (6)$$

Lateral stability equation

$$l > \frac{-C_{y_\beta}(D/W)^2 + 1\{38.67(1 - W_1/W) + r^2/4(W_1/W)\}}{20 C_{n_b}(D/W) - \dfrac{C_{y_b}(\omega_l r^2)^2 (W_1/W)^2}{1372(1 - W_1/W) + 32.2 r^2 (W_1/W)}} \qquad (7)$$

where r is the wheel radius, W_1 the weight of the wheel, and W the total weight of the slung load.

Results

To determine the effect that the wheel has on stability, Eqs. (6) and (7) are first plotted in Figs. 2 and 3 with the weight W_1 set to zero. These results are identical to those obtained by Poli and Cromack for a single-point suspension system. It can be seen that long cables and large drag-to-weight ratios are required for stability. A large drag-to-weight ratio implies high speeds and light loads. Cable lengths increase rapidly at drag-to-weight ratios less than 0.02 and tend to level off at values greater than this. If the conditions for lateral stability are satisfied, then longitudinal stability is guaranteed. When the wheel is incorporated into the system, the curve determined for

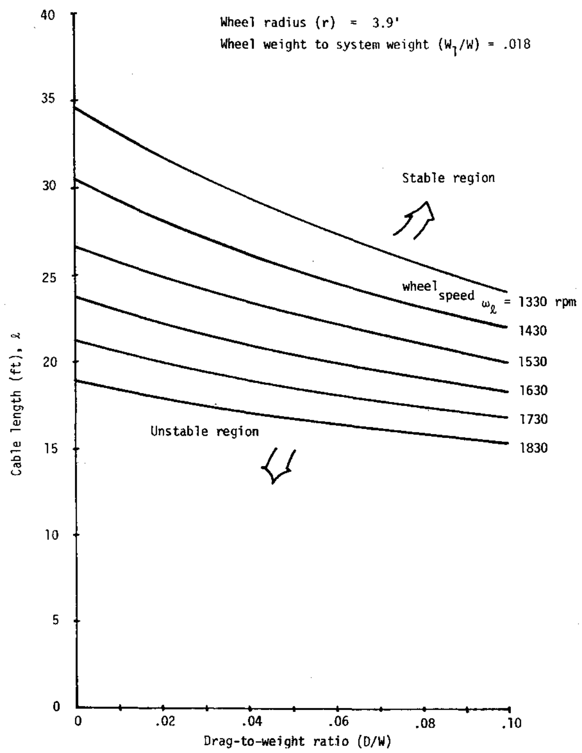

Fig. 5 Lateral stability of system, varying wheel speed.

longitudinal stability in Fig. 2 will not change; however, lateral stability will be greatly affected. Figures 4–6 show the effect that the reaction wheel has on lateral stability. From Fig. 4 it is seen that as the wheel size increases, the cable length required for stability decreases. Similar results are obtained in Figs. 5 and 6 when the wheel speed or weight is increased. These plots show that the cable length can be decreased to almost any length such that lateral stability is no longer the controlling factor, but longitudinal stability is.

While not completely satisfactory, the addition of the reaction wheel is seen to have a tremondous effect on lateral stability. This investigation has given an indication of additional stability parameters, and a possible method of controlling the lateral mode during airborne towing. Further investigations are necessary to determine an effective means of controlling both longitudinal and lateral stability. These investigations are currently underway at the University of Massachusetts.

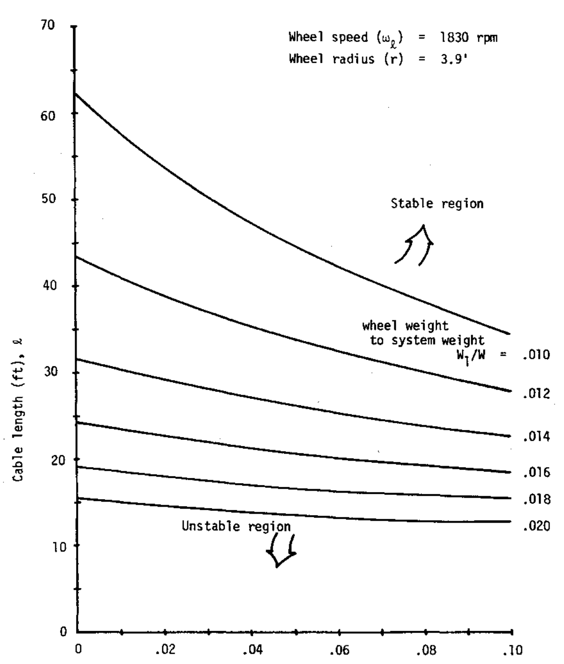

Fig. 6 Lateral stability of system, varying wheel weight.

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On the Fuel Optimality of Cruise

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Introduction

FOR a particular aircraft model where the control variables are thrust and flight path angle, the cruise condition was shown to be a doubly singular arc in the calculus of variations by satisfying the first order necessary conditions. In this note, the singular arc is shown *not* to be minimizing by applying the vector control form of the generalized Legendre-Clebsch condition.

In a paper by Schultz and Zagalsky[1] the minimum fuel-fixed range problem is discussed by considering a particular mathematical model for the aircraft dynamics. In this model thrust and flight path angle are control variables which both assume, within some bounded set, intermediate values during cruise. Cruise is an extremal arc which is called a singular arc in the calculus of variations.[2-4] In Ref. 1 only the first order necessary conditions which generate the singular arc were considered. However, additional second order necessary conditions are available for singular control problems. Here, use is made of the generalized Legendre-Clebsch condition, due first to Kelley,[5] and later generalized to the vector control case by Kelley et al.,[3] Robbins[6] and Goh.[7] It is shown here that the generalized

Received July 24, 1973.
Index categories: Aircraft Performance; Navigation, Control and Guidance Theory.

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